

STEADY STATE SOLUTION OF INTERDEPENDENT FORKED QUEUE OF BULK ARRIVALS WITH LOAD DEPENDENT SERVICE RATE

ATCHUTARAO SADU

Department of Engineering Mathematics, MVGR College of Engineering, Vizianagaram.
dr.atchut.sadu@gmail.com

Abstract:

In this paper a forked two node tandem queueing model with load dependent service rates having bulk arrivals is introduced and analyzed under equilibrium conditions. The system performance measures like probability of emptiness of the system, the probabilities of emptiness of marginal queues, the average number of customers in each queue, the average waiting time of the customer in each queue, the throughput of each node, the utility of servers, and the variance of the number of customers in each queue are derived and analyzed. The sensitivity analysis of a model reveals that the bulk size distribution parameters have a significant influence on system performance measures. Through numerical studies, it is observed that the load dependent service rates can reduce the congestion in queues and mean delay. This model also includes some of the earlier models as particular cases for specific values of the parameters.

Keywords: Load dependent service rate, forked queue, bulk arrivals, Performance Evaluation.

I. INTRODUCTION

Queueing models create lot of interest due to their ready applicability. Several queueing models have been developed and analyzed with various assumptions in order to accommodate many practical situations arising at places like communication networks, transportation systems, production processes, etc. In many of the realistic situations the output from one queueing process servers as an input to another. This sort of queueing systems are known as tandem or serial queueing systems. Tandem queueing models provide basic framework for design and monitoring of communication networks, production processes, assembly lines, etc. For example, in store and forwarded systems the messages arrive to the source and converted into a random number of packets and stored in buffer for forward transmission. After getting transmission from first transmitter, the packets are split into two categories namely, data and voice packets.

All the data packets are routed through a buffer connected to the second transmitter and all the voice packets are routed through a third buffer connected to third transmitter which is in parallel to the second buffer. This type of queueing systems are known as Forked queueing systems. A queue is a waiting line of units demanding service at a service facility. The mathematical representation of a queueing system is known as a queueing model. Queueing models create lot of interest due to their ready applicability in wide varieties of situations prevailing at several places like Communication networks, Transportation systems, Production processes, Biological and population growth studies, Inventory control, Machine repairing, Reliability analysis, Computer performance evaluation Internet, Traffic flow control, ATM scheduling, Tele and satellite communications.

In store and forwarded communication networks the messages arrived at the sources are converted into a random number of packets depending upon the size of the message. Similarly, in cargo handling at rail yards and ports the goods arrive for loading or unloading through the wagons / vessels containing random number of units. In both the situations the arrival of units to the system is in bulk having modules of random sizes. These bulk arrivals can be characterized with compound Poisson process. The compound Poisson process also includes Poisson process as a particular case.

O' Brien, G.G. (1954), Jackson, R.R.P. (1954, 1956), Jackson, J.R. (1957) have pioneered the tandem queueing models which were strengthened by the scientific arguments of Paul J. Burke (1956) the output process distribution. Several authors have studied various queueing models with various assumptions.

Yukuo Hayashida (1993) studied the tandem type go back N scheme satellite communication to reduce the long round trip delay. Peter G. Harrison and Edwige Pitel (1995) obtained the Laplace transform of the probability distribution of the end – to – end delay in tandem networks. Baris Tan (1997) studied the variance of the throughput of an N-station production line with no intermediate buffers and time dependent failures is analytically determined. Debicki, K.G., Mandjes, M.R.H. and Van Uitert, M.J.G. (2006) presented the steady state distribution of the workload of the second queue in a two-node tandem network. Dudin, A.N., Kim, C.S., Klimenok, V.I. and Taramin, O.S. (2010) analyzed a dual tandem queue having a single server queueing system with infinite buffer at the first station and a multi-server queueing system with a finite buffer at the second station. Wenhui Zhou and Zhaotong Lian (2011) analyzed a two-stage tandem network. The customers waiting in these two stages share one finite buffer. Parthasarathy, P.R. and Selvaraju, N. (2001), Kin K. Leung (2002), Suresh Varma, P. and Srinivasarao, K. (2007), Padmavathi, G., Srinivasarao, K and Reddy, K.V.V.S (2009) have developed queueing models with load dependent (state dependent) service rates. However, they assumed that the arrivals to the system are in single and can be characterized by Poisson process. Hence, in this paper a three-node forked queueing model with load dependent service rates having bulk arrivals is developed and analyzed. Here, it is assumed that in each node the transmission rate is dependent on the content of the queue connected to it. The service processes in three service stations follow Poisson processes. It is further assumed that the customers arrive in batches of random size and the

arrival process formulate a compound Poisson process with parameter λ and batch size distribution $\{C_x, x=1,2,3,\dots\}$.

Using Steady -State equations, the equilibrium behavior of the model is analyzed by deriving the joint probability generating function of the number of customers in each queue. The system performance measures like probability of emptiness of the system, the probabilities of emptiness of marginal queues, the average number of customers in each queue, the average waiting time of the customer in each queue, the utilization of the servers the throughput of each node, the variance of the number of customers in each queue are derived explicitly.

The operating characteristics of the model with respect to the parameters is also presented through numerical illustrations by assuming that the number of customers in each arriving module follows a Uniform distribution. The steady state behavior of the model is also studied by deriving the performance measures of the system as a limiting case.

II. STEADY STATE SOLUTION OF THE QUEUEING MODEL

Consider a queueing system in which three queues are connected in tandem in which the customers arrive to the first queue in bulk. After getting service from the first service station, the customer may join the queue connected to the second or third service stations which are in parallel with certain probabilities.

Let Q_1 , Q_2 and Q_3 be the three queues and S_1 , S_2 and S_3 are the service stations in order i.e., the customers after getting service at S_1 may join either in the second queue, Q_2 with probability π or in the third queue, Q_3 with probability $\theta (=1-\pi)$. It is further assumed that the customers arrive to the first queue in batches of random size. i.e., the actual number of customers in any arriving module is a random variable X with probability C_x . If λ_x is the arrival rate of the Poisson process of batches of size X then $C_x = \frac{\lambda_x}{\lambda}$ where, λ is composite arrival rate of all batches and is equal to $\sum \lambda_i$. This implies the arrival of customers follows a compound Poisson process with arrival rate λ having bulk size distribution $\{C_x, x=1,2,3,\dots\}$. The number of service completions in all three service stations follow poisson processes with parameters μ_1 , μ_2 and μ_3 of first, second and third service stations respectively. It is further assumed that the service rates in each service station are linearly dependent on the content of the queue connected to it. The queue discipline is first in first out. The schematic diagram representing the queuing model is shown in Figure.

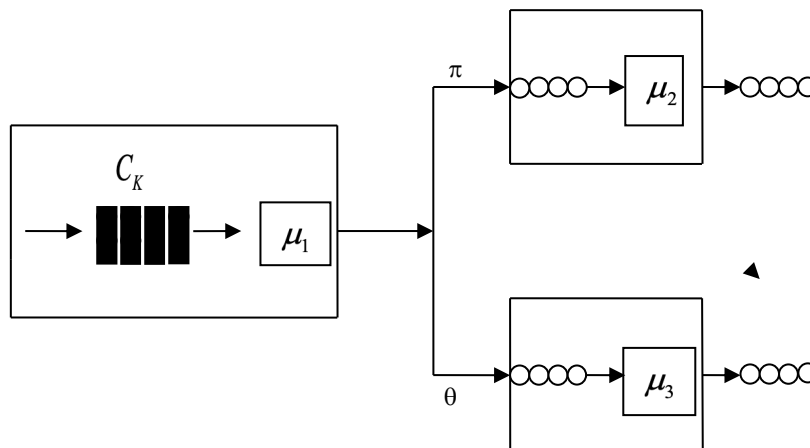


Figure: Schematic diagram of the queuing model

Let n_1 , n_2 and n_3 denote the number of customers in first, second and third queues respectively, and $P_{n_1, n_2, n_3}(t)$ be the probability that there are n_1 customers in the first queue, n_2 customers in the second queue and n_3 customers in the third queue at time t .

The steady-state equations governing the model are

$$-(\lambda + n_1\mu_1 + n_2\mu_2 + n_3\mu_3)P_{n_1, n_2, n_3} + \lambda \left[\sum_{k=1}^{\infty} C_k P_{n_1-k, n_2, n_3} \right] + (n_1 + 1)\pi\mu_1 P_{n_1+1, n_2-1, n_3}$$

$$\begin{aligned}
 & + (n_1 + 1)\theta\mu_1 P_{n_1+1,n_2,n_3-1} + (n_2 + 1)\mu_2 P_{n_1,n_2+1,n_3} + (n_3 + 1)\mu_3 P_{n_1,n_2,n_3+1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 > 0, n_2 > 0, n_3 > 0 \\
 & - (\lambda + n_2\mu_2 + n_3\mu_3)P_{0,n_2,n_3} + \pi\mu_1 P_{1,n_2-1,n_3} + \theta\mu_1 P_{1,n_2,n_3-1} \\
 & \quad + (n_2 + 1)\mu_2 P_{0,n_2+1,n_3} + (n_3 + 1)\mu_3 P_{0,n_2,n_3+1} = 0 \qquad \text{for } n_1 = 0, n_2 > 0, n_3 > 0 \\
 & - (\lambda + n_1\mu_1 + n_3\mu_3)P_{n_1,0,n_3} + \lambda \left[\sum_{k=1}^{\infty} C_k P_{n_1-k,0,n_3} \right] \\
 & \quad + (n_1 + 1)\theta\mu_1 P_{n_1+1,0,n_3-1} + \mu_2 P_{n_1,1,n_3} + (n_3 + 1)\mu_3 P_{n_1,0,n_3+1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 > 0, n_2 = 0, n_3 > 0 \\
 & - (\lambda + n_1\mu_1 + n_2\mu_2)P_{n_1,n_2,0} + \lambda \left[\sum_{k=1}^{\infty} C_k P_{n_1-k,n_2,0} + (n_1 + 1)\pi\mu_1 P_{n_1+1,n_2-1,0} \right] \\
 & \quad + (n_2 + 1)\mu_2 P_{n_1,n_2+1,0} + \mu_3 P_{n_1,n_2,0} = 0 \qquad \text{for } n_1 > 0, n_2 > 0, n_3 = 0 \\
 & - (\lambda + n_3\mu_3)P_{0,0,n_3} + \theta\mu_1 P_{1,0,n_3-1} + \mu_2 P_{0,1,n_3} + (n_3 + 1)\mu_3 P_{0,0,n_3+1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 = 0, n_2 = 0, n_3 > 0 \\
 & - (\lambda + n_1\mu_1)P_{n_1,0,0} + \lambda \left[\sum_{k=1}^{\infty} C_k P_{n_1-k,0,0} \right] + \mu_2 P_{n_1,1,0} + \mu_3 P_{n_1,0,1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 > 0, n_2 = 0, n_3 = 0 \\
 & - (\lambda + n_2\mu_2)P_{0,n_2,0} + \pi\mu_1 P_{1,n_2-1,0} + (n_2 + 1)\mu_2 P_{0,n_2+1,0} + \mu_3 P_{0,n_2,1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 = 0, n_2 > 0, n_3 = 0 \\
 & - \lambda P_{0,0,0} + \mu_2 P_{0,1,0} + \mu_3 P_{0,0,1} = 0 \\
 & \qquad \qquad \qquad \text{for } n_1 = 0, n_2 = 0, n_3 = 0
 \end{aligned}$$

(1)

Let the joint probability generating function is

$$P(z_1, z_2, z_3) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} z_1^{n_1} z_2^{n_2} z_3^{n_3} P_{n_1,n_2,n_3} \quad (2)$$

Multiplying the above of equations with $z_1^{n_1}, z_2^{n_2}$ and $z_3^{n_3}$ and summing over n_1, n_2 , and n_3 then we get

$$\mu_1(\pi z_2 + \theta z_3 - z_1) \frac{\partial P}{\partial z_1} + \mu_2(1 - z_2) \frac{\partial P}{\partial z_2} + \mu_3(1 - z_3) \frac{\partial P}{\partial z_3} + \lambda P(1 - C(z_1)) = 0$$

$$\text{where, } C(z_1) = \sum_{k=1}^{\infty} z_1^k c_k \text{ and } P = P(z_1, z_2, z_3) \quad (3)$$

Solving the equation (3), by Lagrange's method, auxiliary equations are

$$0 = \frac{dz_1}{\mu_1(z_1 - \pi z_2 - \theta z_3)} = \frac{dz_2}{\mu_2(z_2 - 1)} = \frac{dz_3}{\mu_3(z_3 - 1)} = \frac{dP}{\lambda P(C(z_1) - 1)} \quad (4)$$

Therefore joint probability generating function is obtained as

$$P(z_1, z_2, z_3) = \exp \left\{ \lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r \sum_{j=0}^i (-1)^i C_k \binom{k}{r} \binom{r}{i} \binom{i}{j} \right. \\ \left. \left[z_1 - \pi - \theta + \pi \mu_1 \left(\frac{z_2 - 1}{\mu_2 - \mu_1} \right) + \theta \mu_1 \left(\frac{z_3 - 1}{\mu_3 - \mu_1} \right) \right]^{r-i} \right. \\ \left. \pi \mu_1 \left[\left(\frac{z_2 - 1}{\mu_2 - \mu_1} \right) \right]^{i-j} \left[\theta \mu_1 \left(\frac{z_3 - 1}{\mu_3 - \mu_1} \right) \right]^j \left[\frac{1}{(j \mu_3 + (i-j) \mu_2 + (r-i) \mu_1)} \right] \right\} \\ \lambda < \min \{ \mu_1, \mu_2, \mu_3 \} \quad (5)$$

Expanding $P(z_1, z_2, z_3)$ given in equation (5) and collecting the constant terms, we get the probability that the system is empty as

$$P_{0,0,0} = \exp \left\{ \lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r \sum_{j=0}^i (-1)^{r+i} C_k \binom{k}{r} \binom{r}{i} \binom{i}{j} \left[1 + \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right) + \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right) \right]^{r-i} \right. \\ \left. \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right)^{i-j} \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right)^j \left(\frac{1}{j \mu_3 + (i-j) \mu_2 + (r-i) \mu_1} \right) \right\}; \\ \lambda < \min \{ \mu_1, \mu_2, \mu_3 \} \quad (6)$$

Taking $z_2 = 1, z_3 = 1; z_1 = 1, z_3 = 1$ and $z_1 = 1, z_2 = 1$ in $P(z_1, z_2, z_3)$, the probability generating functions of the m^{th} queue size are obtained respectively as

$$P(z_m) = \begin{cases} \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k C_k \binom{k}{r} (z_1 - 1)^r \left(\frac{1}{r \mu_1} \right) \right], & \text{for } m = 1 \\ \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r C_k \binom{k}{r} \binom{r}{i} (-1)^i \left(\frac{\pi \mu_1 (z_2 - 1)}{\mu_2 - \mu_1} \right)^r \left(\frac{1}{i \mu_2 + (r-i) \mu_1} \right) \right], & \text{for } m = 2 \\ \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{j=0}^r C_k \binom{k}{r} \binom{r}{j} (-1)^j \left(\frac{\theta \mu_1 (z_3 - 1)}{\mu_3 - \mu_1} \right)^r \left(\frac{1}{j \mu_3 + (r-j) \mu_1} \right) \right], & \text{for } m = 3 \end{cases} \quad (7)$$

Expanding $P(z_m)$, for $m = 1, 2, 3$ and collecting the constant terms, we get the probability that the first, second and third queues are respectively empty as

$$P_{0,.,.} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k C_k \binom{k}{r} \frac{(-1)^r}{r \mu_1} \right] \\ P_{.,0,.} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r C_k \binom{k}{r} \binom{r}{i} (-1)^{i+r} \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right)^r \left(\frac{1}{i \mu_2 + (r-i) \mu_1} \right) \right] \\ P_{.,.,0} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{j=0}^r C_k \binom{k}{r} \binom{r}{j} (-1)^{j+r} \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right)^r \left(\frac{1}{(j \mu_3 + (r-j) \mu_1)} \right) \right] \quad (8)$$

The mean number of customers in the m^{th} queue is

$$L_m = \begin{cases} \left(\frac{\lambda}{\mu_1} \right) E(X), & \text{for } m = 1 \\ \left(\frac{\lambda \pi}{\mu_2} \right) E(X), & \text{for } m = 2 \\ \left(\frac{\lambda \theta}{\mu_3} \right) E(X), & \text{for } m = 3 \end{cases} \quad (9)$$

The utilization of the first, second and third servers are

$$U_1 = 1 - P_{0, \dots, \dots};$$

$$U_2 = 1 - P_{\dots, 0, \dots} \text{ and}$$

$$U_3 = 1 - P_{\dots, \dots, 0} \quad (10)$$

The Throughput of the m^{th} queue is

$$Thp_m = \mu_m U_m, \text{ for } m = 1, 2, 3 \quad (11)$$

The average delay in the m^{th} queue is

$$W_m = \frac{L_m}{Thp_m}, \text{ for } m = 1, 2, 3$$

The variance of number of customers in the m^{th} queue is

$$V_m = \begin{cases} \frac{\lambda}{2\mu_1} \sum_{k=1}^{\infty} [k^2 C_k + k C_k], & \text{for } m = 1 \\ \frac{\lambda \pi^2 \mu_1}{\mu_2 (\mu_1 + \mu_2)} \sum_{k=2}^{\infty} \binom{k}{2} C_k + \frac{\lambda \pi}{\mu_2} E(X), & \text{for } m = 2 \\ \frac{\lambda \theta^2 \mu_1}{\mu_3 (\mu_1 + \mu_3)} \sum_{k=2}^{\infty} \binom{k}{2} C_k + \frac{\lambda \theta}{\mu_3} E(X), & \text{for } m = 3 \end{cases} \quad (13)$$

III. PERFORMANCE MEASURES WITH UNIFORM BATCH SIZE DISTRIBUTION

The performance of the queuing model is highly influenced by the form of batch size distribution. It is assumed that the number of customers in any arriving module is a random variable and it follows a uniform distribution with parameters a and b , then the probability mass function of the batch size arriving module distribution is

$$C_k = \frac{1}{b - a + 1}, \quad \text{for } k = a, a + 1, \dots, b \quad (14)$$

$$\text{The mean number of customers in a batch is } L = \frac{a + b}{2} \quad (15)$$

$$\text{The variance of batch size is } V = \frac{1}{12} [(b - a + 1)^2 - 1] \quad (16)$$

The joint probability generating function is obtained as

$$P(z_1, z_2, z_3) = \exp \left\{ \lambda \sum_{k=a}^b \sum_{r=1}^k \sum_{i=0}^r \sum_{j=0}^i (-1)^i \left(\frac{1}{b-a+1} \right) \binom{k}{r} \binom{r}{i} \binom{i}{j} \right. \\ \left[z_1 - 1 + \left(\frac{\pi \mu_1 (z_2 - 1)}{\mu_2 - \mu_1} \right) + \left(\frac{\theta \mu_1 (z_3 - 1)}{\mu_3 - \mu_1} \right) \right]^{r-i} \left[\frac{\pi \mu_1 (z_2 - 1)}{\mu_2 - \mu_1} \right]^{i-j} \\ \left. \left[\frac{\theta \mu_1 (z_3 - 1)}{\mu_3 - \mu_1} \right]^j \left[\frac{1}{j \mu_3 + (i-j) \mu_2 + (r-i) \mu_1} \right] \right\}; \\ \lambda < \min \{ \mu_1, \mu_2, \mu_3 \} \quad (17)$$

The probability that the system is empty is

$$P_{0,0,0} = \exp \left\{ \lambda \sum_{k=a}^b \sum_{r=1}^k \sum_{i=0}^r \sum_{j=0}^i (-1)^{r+i} \left(\frac{1}{b-a+1} \right) \binom{k}{r} \binom{r}{i} \binom{i}{j} \left[1 + \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right) + \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right) \right]^{r-i} \right. \\ \left. \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right)^{i-j} \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right)^j \left[\frac{1}{j \mu_3 + (i-j) \mu_2 + (r-i) \mu_1} \right] \right\}; \\ \lambda < \min \{ \mu_1, \mu_2, \mu_3 \} \quad (18)$$

The probability generating functions of the m^{th} queue is obtained respectively as

$$P(z_m) = \begin{cases} \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \left(\frac{1}{b-a+1} \right) \binom{k}{r} (z_1 - 1)^r \left(\frac{1}{r \mu_1} \right) \right], & \text{for } m = 1 \\ \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r \left(\frac{(-1)^i}{b-a+1} \right) \binom{k}{r} \binom{r}{i} \left(\frac{\pi \mu_1 (z_2 - 1)}{\mu_2 - \mu_1} \right)^r \left(\frac{1}{i \mu_2 + (r-i) \mu_1} \right) \right], & \text{for } m = 2 \\ \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{j=0}^r \left(\frac{(-1)^j}{b-a+1} \right) \binom{k}{r} \binom{r}{j} \left(\frac{\theta \mu_1 (z_3 - 1)}{\mu_3 - \mu_1} \right)^r \left(\frac{1}{j \mu_3 + (r-j) \mu_1} \right) \right], & \text{for } m = 3 \end{cases} \quad (19)$$

Expanding $P(z_m)$, for $m = 1, 2, 3$ and collecting the constant terms, we get the probability that the first, second and third queues are respectively empty as

$$P_{0,\dots} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \left(\frac{1}{b-a+1} \right) \binom{k}{r} \frac{(-1)^r}{r \mu_1} \right] \\ P_{\dots,0} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{i=0}^r \left(\frac{(-1)^{i+r}}{b-a+1} \right) \binom{k}{r} \binom{r}{i} \left(\frac{\pi \mu_1}{\mu_2 - \mu_1} \right)^r \left(\frac{1}{i \mu_2 + (r-i) \mu_1} \right) \right] \\ P_{\dots,0} = \exp \left[\lambda \sum_{k=1}^{\infty} \sum_{r=1}^k \sum_{j=0}^r \left(\frac{(-1)^{j+r}}{b-a+1} \right) \binom{k}{r} \binom{r}{j} \left(\frac{\theta \mu_1}{\mu_3 - \mu_1} \right)^r \left(\frac{1}{j \mu_3 + (r-j) \mu_1} \right) \right] \quad (20)$$

The mean number of customers in the m^{th} queue is

$$L_m = \begin{cases} \frac{\lambda(a+b)}{2\mu_1}, & \text{for } m=1 \\ \frac{\lambda\pi(a+b)}{2\mu_2}, & \text{for } m=2 \\ \frac{\lambda\theta(a+b)}{2\mu_3}, & \text{for } m=3 \end{cases} \quad (21)$$

The utilization of the first, second and third queues are

$$U_1 = 1 - P_{0,\dots}, U_2 = 1 - P_{\dots,0}, \text{ and } U_3 = 1 - P_{\dots,0} \quad (22)$$

The Throughput of the m^{th} queue is

$$Thp_m = \mu_m U_m, \text{ for } m=1,2,3 \quad (23)$$

The average delay in the m^{th} queue is

$$W_m = \frac{L_m}{Thp_m}, \text{ for } m=1,2,3 \quad (24)$$

The variance of number of customers in the m^{th} queue is

$$V_m = \begin{cases} \frac{\lambda}{2\mu_1} \left[\sum_{k=a}^b \left(\frac{k^2}{b-a+1} \right) + \frac{a+b}{2} \right], & \text{for } m=1 \\ \frac{\lambda\pi^2\mu_1}{\mu_2(\mu_1+\mu_2)} \sum_{k=a}^b \binom{k}{2} \left(\frac{1}{b-a+1} \right) + \frac{\lambda\pi(a+b)}{2\mu_2}, & \text{for } m=2 \\ \frac{\lambda\theta^2\mu_1}{\mu_3(\mu_3+\mu_1)} \sum_{k=a}^b \binom{k}{2} \left(\frac{1}{b-a+1} \right) + \frac{\lambda\theta(a+b)}{2\mu_3}, & \text{for } m=3 \end{cases} \quad (25)$$

IV. PERFORMANCE EVALUATION OF THE QUEUEING MODEL

In this section, the performance of the proposed queueing model is discussed through numerical illustration.

Since performance characteristics of queueing model are sensitive with respect to the parameters, the steady state behavior of model is studied through computing the performance measures with the following set of values for the model parameters:

$a = 1, 2, 3, 4, 5$; $b = 10, 15, 20, 25, 30$; $\lambda = 1.5, 2.0, 2.5, 3.0, 3.5$; $\mu_1 = 3, 4, 5, 6, 7$; $\mu_2 = 8, 9, 11, 12, 13$; $\mu_3 = 8, 12, 16, 22, 25$; and $\pi = 0.1, 0.3, 0.5, 0.7, 0.9$.

For different values of the parameters the values of probability of emptiness of each queue, the average number of customers in each queue, the throughput of each node, the mean waiting time of a customer in each queue are computed and presented in **Table**.

From **Table**, it is observed that the batch size distribution parameters have significant influence on the performance measures. As a and b are increasing the probability of emptiness of each queue is decreasing, and the average number of customers in the queue and the average waiting time of a customer are increasing when other parameters are fixed.

Table 3. PERFORMANCE OF OPERATING CHARACTERISTICS OF SYSTEM FOR DIFFERENT VALUES OF THE PARAMETERS

a	b	λ	μ_1	μ_2	μ_3	π	$P_{0,0,0}$	$P_{0,\dots}$	$P_{\dots,0}$	$P_{\dots,0}$	L_1	L_2	L_3	Thp_1	W_1	Thp_2	W_2	Thp_3	W_3
1	25	2	4	10	20	0.4	0.1912	0.2267	0.4939	0.5750	6.50	1.04	0.780	3.0934	2.1013	5.0606	0.2055	8.4992	0.0918
2	25	2	4	10	20	0.4	0.1833	0.2176	0.4812	0.5633	6.75	1.08	0.810	3.1298	2.1567	5.1876	0.2082	8.7332	0.0927
3	25	2	4	10	20	0.4	0.1771	0.2103	0.4693	0.5523	7.00	1.12	0.840	3.1586	2.2161	5.3066	0.2111	8.9548	0.0938
4	25	2	4	10	20	0.4	0.1720	0.2043	0.4582	0.5418	7.25	1.16	0.870	3.1828	2.2778	5.4183	0.2141	9.1649	0.0949
5	25	2	4	10	20	0.4	0.1675	0.1990	0.4477	0.5318	7.50	1.20	0.900	3.2038	2.3409	5.5223	0.2173	9.3644	0.0961
5	10	2	4	10	20	0.4	0.2259	0.2680	0.6081	0.6827	3.75	0.60	0.450	2.9281	1.2807	3.9186	0.1531	6.3463	0.0709
5	15	2	4	10	20	0.4	0.1996	0.2370	0.5404	0.6210	5.00	0.80	0.600	3.0521	1.6382	4.5960	0.1741	7.5800	0.0792
5	20	2	4	10	20	0.4	0.1813	0.2153	0.4885	0.5718	6.25	1.00	0.750	3.1386	1.9913	5.1147	0.1955	8.5634	0.0876
5	25	2	4	10	20	0.4	0.1675	0.1990	0.4477	0.5318	7.50	1.20	0.900	3.2038	2.3409	5.5233	0.2173	9.3644	0.0961
5	30	2	4	10	20	0.4	0.1566	0.1861	0.4147	0.4986	8.75	1.40	0.050	3.2555	2.6878	5.8528	0.2392	10.029	0.1047
5	25	1.5	4	10	20	0.4	0.2618	0.2980	0.5473	0.6227	5.62	0.90	0.675	2.8080	4.0032	4.5271	0.1988	7.5454	0.0895
5	25	2.0	4	10	20	0.4	0.1675	0.1990	0.4477	0.5318	7.50	1.20	0.900	3.2038	2.3409	5.5233	0.2173	9.3644	0.0961
5	25	2.5	4	10	20	0.4	0.1072	0.1329	0.3662	0.4541	9.37	1.50	1.125	3.4682	2.7031	6.3382	0.2367	10.917	0.1030
5	25	3.0	4	10	20	0.4	0.0686	0.0888	0.2995	0.3878	11.2	1.80	1.350	3.6448	3.0866	7.0047	0.2570	12.244	0.1103
5	25	3.5	4	10	20	0.4	0.0440	0.0593	0.245	0.3312	13.1	2.10	1.575	3.7627	3.4881	7.5499	0.2781	13.376	0.1177
5	25	2.0	3	10	20	0.4	0.0988	0.1162	0.4229	0.5083	10.0	1.20	0.900	2.6514	3.7714	5.7714	0.2079	9.8339	0.0915
5	25	2.0	4	10	20	0.4	0.1675	0.1990	0.4477	0.5318	7.50	1.20	0.900	3.2038	2.3409	5.5233	0.2173	9.3644	0.0961
5	25	2.0	5	10	20	0.4	0.2289	0.2749	0.4679	0.5519	6.00	1.20	0.900	3.6256	1.6549	5.3205	0.2255	8.9628	0.1004
5	25	2.0	6	10	20	0.4	0.2807	0.3409	0.4847	0.5692	5.00	1.20	0.900	3.9546	1.2644	5.1525	0.2329	8.6159	0.1045
5	25	2.0	7	10	20	0.4	0.3237	0.3976	0.4988	0.5843	4.28	1.20	0.900	4.2171	1.0163	5.0116	0.2394	8.3136	0.1083
5	25	2.0	4	8	20	0.4	0.1609	0.1990	0.3870	0.5318	7.50	1.50	0.900	3.2038	2.3409	4.9037	0.3059	9.3644	0.0961
5	25	2.0	4	9	20	0.4	0.1647	0.1990	0.4191	0.5318	7.50	1.33	0.900	3.2038	2.3409	5.2282	0.2550	9.3644	0.0961
5	25	2.0	4	11	20	0.4	0.1697	0.1990	0.4734	0.5318	7.50	1.09	0.900	3.2038	2.3409	5.7931	0.1883	9.3644	0.0961
5	25	2.0	4	12	20	0.4	0.1714	0.1990	0.4966	0.5318	7.50	1.00	0.900	3.2038	2.3409	6.0409	0.1655	9.3644	0.0961
5	25	2.0	4	13	20	0.4	0.1728	0.1990	0.5177	0.5318	7.50	0.92	0.900	3.2038	2.3409	6.2694	0.1472	9.3644	0.0961
5	25	2.0	4	10	8	0.4	0.1468	0.1990	0.4477	0.3023	7.50	1.20	2.250	3.2038	2.3409	5.5233	0.2173	5.5813	0.4031
5	25	2.0	4	10	12	0.4	0.1593	0.1990	0.4477	0.4028	7.50	1.20	1.500	3.2038	2.3409	5.5233	0.2173	7.1664	0.2093
5	25	2.0	4	10	16	0.4	0.1646	0.1990	0.4477	0.4756	7.50	1.20	1.125	3.2038	2.3409	5.5233	0.2173	8.3896	0.1341
5	25	2.0	4	10	22	0.4	0.1686	0.1990	0.4477	0.5554	7.50	1.20	0.818	3.2038	2.3409	5.5233	0.2173	9.7812	0.0836
5	25	2.0	4	10	25	0.4	0.1697	0.1990	0.4477	0.5866	7.50	1.20	0.720	3.2038	2.3409	5.5233	0.2173	10.336	0.0697
5	25	2.0	4	10	20	0.1	0.1752	0.1990	0.7649	0.4395	7.50	0.30	1.350	3.2038	2.3409	2.3510	0.1276	11.210	0.1204
5	25	2.0	4	10	20	0.3	0.1700	0.1990	0.5176	0.496	7.50	0.90	1.050	3.2038	2.3409	4.8237	0.1866	10.080	0.1042
5	25	2.0	4	10	20	0.5	0.1651	0.1990	0.3962	0.5745	7.50	1.50	0.750	3.2038	2.3409	6.0375	0.2484	8.5106	0.0881
5	25	2.0	4	10	20	0.7	0.1607	0.1990	0.3264	0.6894	7.50	2.10	0.450	3.2038	2.3409	6.7356	0.3118	6.2120	0.0724
5	25	2.0	4	10	20	0.9	0.1566	0.1990	0.2816	0.8691	7.50	2.70	0.150	3.2038	2.3409	7.1843	0.3758	2.6182	0.0573

This model also includes some of the earlier models as particular cases for specific values of the parameters. These models are much useful for performance evaluation of communication networks.

V. CONCLUSION

In this paper a novel and new queueing model which is useful for performance evaluations of several communication networks, production processes, Assembly systems, etc is developed and analyzed.

Here it is assumed that the arrivals are in bulk of random size. It is further assumed the service rate at each service station is linearly dependent on the content of the connected to it.

The system performance measures like probability of emptiness of the system, the probabilities of emptiness of marginal queues, the average number of customers in each queue, the average waiting time of the customer in each queue, the throughput of each node, the utility of servers, and the variance of the number of customers in each queue are derived and analyzed explicitly.

So regulating the input parameters one can control the congestion in queues and avoid burst ness of the lines.

This model also includes some of the earlier models in tandem as particular cases for specific values of the parameters. In this paper the bulk size distribution is considered as uniform.

It is also possible to analyze the model with binomial and truncated geometric bulk size distributions. This model can also be further extended for considering cost aspects and optimization which will be taken as elsewhere.

REFERENCES

1. Benjamin Avi- Itzhak and Hanoch Levy (2001), Buffer Requirement and service ordering in a tandem queue with correlated service times, *Mathematics of Operations Research*, Vol.26, No.2 pp 358-374.
2. Debicki, K.G., Mandjes, M.R.H. and Van Uitert, M.J.G. (2006), A tandem queue with levy input: a new representation of the downstream queue length, probability, *Networks and Algorithms*, Report PNA – E0603.
3. Dudin, A.N., Kim, C.S., Klimenok, V.I. and Taramin, O.S. (2010), A dual tandem queueing system with a finite intermediate buffer and cross traffic, *Proceedings of the 5th International Conference on Queueing Theory and Network Applications*, ACM.
4. Jackson, J.R. (1957), Networks of waiting lines, *Operations Research*, Vol. 5, No.4, pp 518-521.
5. Jackson, R.R.P. (1954), Queueing systems with phase type service, *Operations Research Quart.*, Vol.5, No.4, pp 109-120.
6. Kin K. Leung (2002), Load-dependent service queues with application to congestion control in broadband networks, *Performance Evaluation*, Vol. 50, No.4, pp 27-40.
7. Nageswararao, K., Srinivasarao, K and Srinivasarao, P (2011), A tandem communication network with dynamic bandwidth allocation and modified phase type transmission having bulk arrivals, *International Journal of Computer science Issues*, Vol.7, No.2, pp 18-26.
8. O' Brien, G.G.(1954), The solution of some queueing systems, *J. Soc. Ind. Appl. Math*, Vol.2, pp 132-142.
9. Padmavathi, G., Srinivasarao, K and Reddy, K.V.V.S(2009), Performance evaluation of parallel and series communication network with dynamic bandwidth allocation CIIT *International Journal of networking and communication*, Vol.1, No.7, pp 410-421.
10. Parthasarathy, P.R. and Selvaraju, N. (2001), Transient analysis of a queue where potential customers are discouraged by queue length, *Mathematical Problems in Engineering*, Vol.7, pp 433-454.
11. Paul J. Burke, (1956), The output of a queueing System, *J. Operation Research*, Vol.4. No.6, pp 699-704.
12. Peter G. Harrison and Edwige Pitel (1995), Response time distribution in tandem G-networks, *J. Applied Probability*, Vol. 32, No. 1, pp 224 - 226.
13. Suresh Varma, P. and Srinivasarao, K. (2007), A Communication Network with load dependent transmission , *International Journal of Mathematical Science*, Vol.7,No.2, PP199-210.
14. Wenhui Zhou and Zhaotong Lian (2011), A tandem network with a sharing buffer, *Applied Mathematical Modelling*, Vol.35, No.9, pp 4507-4515.
15. Yukuo Hayashida (1993), Throughput analysis of tandem type go – back – NARQ scheme for satellite communications, *IEEE Transactions on Communications*, Vol. 41, No.10, pp 1517- 1524.